Anomalous scaling dimensions and stable charged fixed-point of type-II superconductors

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The critical properties of a type-II superconductor model are investigated using a dual vortex representation. Computing the propagators of gauge field $\mathbf{A}$ and dual gauge field $\mathbf{h}$ in terms of a vortex correlation function, we obtain the values $\eta_{\mathbf{A}} = 1$ and $\eta_{\mathbf{h}} = 1$ for their anomalous dimensions. This provides support for a dual description of the Ginzburg-Landau theory of type-II superconductors in the continuum limit, as well as for the existence of a stable charged fixed point of the theory, not in the $3DXY$ universality class.

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Determining the universality class of the phase-transition in a system of a charged scalar field coupled to a massless gauge field, such as a type-II superconductor, has been a long-standing problem[1]. Analytical and numerical efforts have recently focused on the use of a *dual* description of the Ginzburg-Landau theory (GLT) of type-II superconductors, pioneered by Kleinert[2], in investigating the character of a proposed novel *stable* fixed point of the theory for a charged superconducting condensate, in which case the $3DXY$ fixed point of the neutral superfluid is rendered unstable[3–6]. The dual formulation has also been employed to investigate the possibility of novel broken symmetries in the vortex liquid phase of such systems in magnetic fields[4,5].

The GLT is defined by a complex matter field ψ coupled to a massless fluctuating gauge field $\mathbf{A}$ with a Hamiltonian

$$H_{\psi,\mathbf{A}} = m_\psi^2 \left|\psi\right|^2 + \frac{u_\psi}{2}\left|\psi\right|^4 + \left|\left(\nabla - i2e\mathbf{A}\right)\psi\right|^2 + \frac{1}{2}\left(\nabla\times\mathbf{A}\right)^2. \quad (1)$$

Here, e is the electron charge, and $H_{\psi,\mathbf{A}}$ is invariant under the local gauge-transformation $\psi \to \psi\exp(i\theta)$, $\mathbf{A} \to \mathbf{A} + \nabla\theta/2ie$. The GLT sustains stable topological objects in the form of vortex lines and vortex loops, the latter are the critical fluctuations of the theory[4,5]. These objects are highly nonlocal in terms of ψ, but a dual formulation offers a local field theory for them. The continuum dual representation of the topological excitations, (in $D = 3$ only), consists of a complex matter field ϕ coupled to a *massive* gauge field $\mathbf{h}$[2], with coupling constant given by the dual charge e_d, and with dual Hamiltonian

$$H_{\phi,\mathbf{h}} = m_\phi^2\left|\phi\right|^2 + \frac{u_\phi}{2}\left|\phi\right|^4 + \left|\left(\nabla - ie_d\mathbf{h}\right)\phi\right|^2 + \frac{1}{2}\left(\nabla\times\mathbf{h}\right)^2 + \frac{1}{2}\left(\nabla\times\mathbf{A}\right)^2 + ie\left(\nabla\times\mathbf{h}\right)\cdot\mathbf{A}. \quad (2)$$

The massiveness of $\mathbf{h}$ reduces the symmetry to a global $U(1)$-invariance. For details on how to obtain this dual Hamiltonian, we refer the reader to the thorough exposition of this presented in the textbook of Kleinert[7]. For $e \neq 0$ the original GLT in Eq. 1 has a local gauge symmetry, the dual theory in Eq. 2 has a global $U(1)$ symmetry. In the limit $e \to 0$, $\mathbf{A}$ decouples from ψ in Eq. 1, H_ψ describes a *neutral superfluid*, and the symmetry is reduced to global $U(1)$. The dual Hamiltonian $H_{\phi,\mathbf{h}}$ describes a charged superfluid coupled to a massless gauge field $\mathbf{h}$ with coupling constant e_d, and the global symmetry is extended to a local gauge symmetry. Hence, when $e \to 0$, *the dual of a neutral superfluid is isomorphic to a superconductor*. Integrating out the $\mathbf{A}$ field in Eq. 2 produces a mass-term $e^2\mathbf{h}^2/2$, where an exact renormalization-group equation for the mass of $\mathbf{h}$ is given by $\partial e^2/\partial\ln l = e^2$[8]. Therefore, when $e \neq 0$, then $e^2 \to \infty$ as $l \to \infty$. This supresses the dual gauge field, and the resulting dual theory is a pure $|\phi|^4$-theory. Hence, in the long-wavelength limit, *the dual of a superconductor is isomorphic to a neutral superfluid*[2].

In this paper, we obtain the anomalous scaling dimensions $\eta_{\mathbf{A}}$ of the gauge field[3,9], as well as $\eta_{\mathbf{h}}$ of the dual gauge field, not previously considered, directly from large-scale Monte-Carlo simulations. At a $3DXY$ critical point,$\eta_{\mathbf{A}} = \eta_{\mathbf{h}} = 0$. We find that ($\eta_{\mathbf{A}} = 1, \eta_{\mathbf{h}} = 0$) when $e \neq 0$, and that ($\eta_{\mathbf{A}} = 0, \eta_{\mathbf{h}} = 1$), when $e = 0$. We also contrast the anomalous dimension of the dual mass field ϕ at the dual charged (original neutral) and dual neutral (original charged) fixed points, obtaining $\eta_\phi = -0.24$ in the former case, and $\eta_\phi = 0.04$ in the latter.

A duality transformation, to a set of interacting vortex loops, is performed on the London/Villain approximation to the GLT. In this approximation the partition function is

$$Z(\beta, e) = \int D\mathbf{A}D\theta \sum_{\{\mathbf{n}\}} \exp\left[-\sum_{\mathbf{x}}\left\{\frac{1}{2}\left(\Delta\times\mathbf{A}\right)^2 + \frac{\beta}{2}\left(\Delta\theta - e\mathbf{A} - 2\pi\mathbf{n}\right)^2\right\}\right]. \quad (3)$$

Here, θ is the local phase of the superconducting order parameter ψ, while $\mathbf{n}$ is an integer-valued *velocity field* (not vortex field) introduced to make the Villain potential 2π-periodic. The symbol Δ denotes a lattice derivative. Amplitude fluctuations are neglected in this ap-

proach. The validity of this approximation for $3D$ systems, has recently been investigated in detail, both numerically and analytically[10].

An auxiliary velocity field $\mathbf{v}$ linearises the kinetic energy. Performing the θ-integration constrains $\mathbf{v}$ to satisfy the condition $\Delta \cdot \mathbf{v} = 0$, explicitly solved by writing $\mathbf{v} = \Delta \times \mathbf{h}$, where $\mathbf{h}$ is forced to integer values by the summation over $\mathbf{n}$. Introducing an integer-valued *vortex field* $\mathbf{m} = \Delta \times \mathbf{n}$, and using Poisson's summation formula, we find

$$S(\mathbf{A},\mathbf{h},\mathbf{m}) = \sum_{\mathbf{x}} \Big\{ 2\pi i \mathbf{m}\cdot\mathbf{h} + \frac{1}{2\beta}(\Delta\times\mathbf{h})^2 + ie(\Delta\times\mathbf{h})\mathbf{A} + \frac{1}{2}(\Delta\times\mathbf{A})^2 \Big\}. \quad (4)$$

Integrating the gauge field in Eq. 4 produces a mass term $e^2\mathbf{h}^2/2$, giving an effective theory containing the vortex field $\mathbf{m}$ coupled to a *massive* gauge field $\mathbf{h}$

$$Z(\beta,e) = \int D\mathbf{h} \sum_{\{\mathbf{m}\}} \prod_{\mathbf{x}} \delta_{\Delta\cdot\mathbf{m},0} \exp\Big[-\sum_{\mathbf{x}} \Big\{ 2\pi i \mathbf{m}\cdot\mathbf{h} + \frac{e^2}{2}\mathbf{h}^2 + \frac{1}{2\beta}(\Delta\times\mathbf{h})^2 \Big\}\Big]. \quad (5)$$

The variables $\mathbf{m}$ in Eq. 5 describe a set of interacting vortices, where the interactions are mediated through the gauge field $\mathbf{h}$. The variables in Eq. 5 are defined on a lattice which is dual to the lattice from Eq. 3, and the behavior with respect to temperature is inverted in the new variables. The θ field in Eq. 3 describes *order*, while the $\mathbf{m}$ field represents the topological excitations of the θ field. These excitations destroy superconducting coherence, and hence quantify *disorder*[7].

Integrating out the $\mathbf{h}$ field in Eq. 5, we obtain the Hamiltonian employed in the present simulations,

$$H(\mathbf{m}) = -2\pi^2 J_0 \sum_{\mathbf{x_1},\mathbf{x_2}} \mathbf{m}(\mathbf{x_1}) V(\mathbf{x_1}-\mathbf{x_2}) \mathbf{m}(\mathbf{x_2}), \quad (6)$$

$$V(\mathbf{x}) = \sum_{\mathbf{q}} \frac{e^{-i\mathbf{q}\cdot\mathbf{x}}}{4\sum_\mu \sin^2\left(\frac{q_\mu}{2}\right) + \lambda^{-2}} . \quad (7)$$

In Eq. 7, the charge e and lattice-spacing a have both been set to unity, and λ is the bare London penetration depth. At every MC step, we attempt to insert a loop of unit vorticity and random orientation. A new energy is calculated from Eq. 6, and the proposed move is accepted or rejected according to the Metropolis algorithm. This procedure ensures that the vortex lines of the system always form closed loops of random size and shape[5]. In all simulations, a system size of $40\times40\times40$ was used, and up to $1.5\cdot10^5$ sweeps over the lattice per temperature were used.

To investigate the properties of $\mathbf{A}$ and $\mathbf{h}$ at the charged critical point of the original theory, Eq. 1, we have calculated the correlation functions $\langle \mathbf{A_q}\mathbf{A_{-q}}\rangle$ and $\langle \mathbf{h_q}\mathbf{h_{-q}}\rangle$ in terms of vortex correlations, obtaining

$$\langle \mathbf{A_q}\mathbf{A_{-q}}\rangle = \frac{1}{|\mathbf{Q}|^2 + m_0^2}\left(1 + \frac{4\pi^2\beta m_0^2 G(\mathbf{q})}{|\mathbf{Q}|^2\left(|\mathbf{Q}|^2 + m_0^2\right)}\right), \quad (8)$$

$$\langle \mathbf{h_q}\mathbf{h_{-q}}\rangle = \frac{2\beta}{|\mathbf{Q}|^2 + m_0^2}\left(1 - \frac{2\beta\pi^2 G(\mathbf{q})}{|\mathbf{Q}|^2 + m_0^2}\right), \quad (9)$$

where $G(\mathbf{q}) = \langle \mathbf{m_q}\mathbf{m_{-q}}\rangle$, $m_0 = \lambda^{-1}$ and $Q_\mu = 1 - e^{-i\mathbf{q}\cdot\hat{\mu}}$. All correlation functions have been calculated in the *transverse gauge* $\nabla\cdot\mathbf{A} = \nabla\cdot\mathbf{h} = 0$. Both of the fields $\mathbf{h}$ and $\mathbf{A}$ are renormalized by vortex fluctuations, albeit in quite different ways.

Invoking the standard form $\left(q^2 + m_{\text{eff}}^2\right)^{-1}$ for the correlation functions in the immediate vicinity of the critical point in the limit $q \to 0$, we find the following expressions for the effective masses,

$$\left(m_{\text{eff}}^{\mathbf{A}}\right)^2 = \lim_{q\to0} \frac{m_0^2}{1 + 4\pi^2\beta G(\mathbf{q}) q^{-2}}, \quad (10)$$

$$\left(m_{\text{eff}}^{\mathbf{h}}\right)^2 = \lim_{q\to0} \frac{m_0^2}{2\beta\left(1 - \frac{2\pi^2\beta G(\mathbf{q})}{m_0^2}\right)}. \quad (11)$$

When $e \neq 0$ the correlation function for $\mathbf{A}$ assumes the form

$$\langle \mathbf{A_q}\mathbf{A_{-q}}\rangle \propto \frac{1}{q^{2-\eta_{\mathbf{A}}}} \quad (12)$$

at the critical point. To determine $\eta_{\mathbf{A}}$, we compute the vortex correlator $G(q)$. For $\lambda << L = 40$, we expect the following behaviour for $G(\mathbf{q})$ in the limit $q \to 0$,

$$T < T_c \Rightarrow G(\mathbf{q}) \propto q^2, \quad (13)$$

$$T = T_c \Rightarrow G(\mathbf{q}) \propto\ q^\eta, \quad (14)$$

$$T > T_c \Rightarrow G(\mathbf{q}) \propto\ C(T). \quad (15)$$

When these limiting forms are inserted in Eq. 10, we see that for $T \leq T_c$, $m_{\text{eff}}^{\mathbf{A}}$ will be finite through the Higgs Mechanism (Meissner effect). For $T \geq T_c$ we will have $m_{\text{eff}}^{\mathbf{A}} = 0$ as in the normal case of a massless photon. Assuming $G(q) \propto q^\eta$ precisely at the critical point, it is seen that η corresponds to $\eta_{\mathbf{A}}$ from Eq. 12. *We thus identify the scaling power of $G(\mathbf{q})$ at the critical point with the anomalous dimension of the massless gauge field* $\mathbf{A}$.

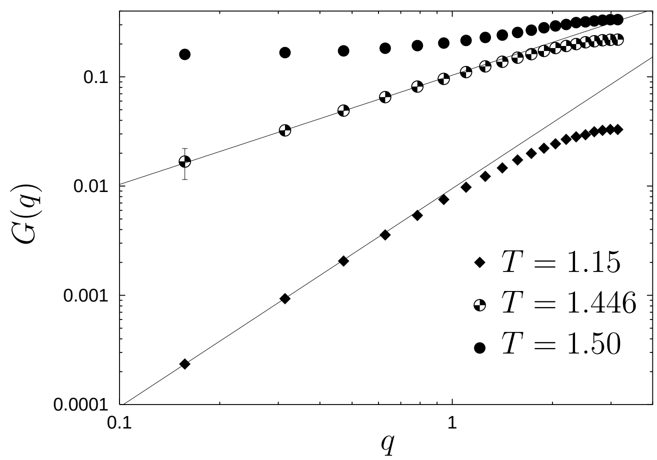

FIG. 1. log-log plot of $G(q)$ for the three alternatives in Eq. 13 - 15, with $\lambda = a/2$. For this λ, $T_c = 1.446$. Apart from the point $q = q_{\min}, T = 1.446$ the error bars are smaller than the symbols used.

All three limiting forms Eqs. 13-15 are shown in Fig. 1. The gauge field masses $m_{\rm eff}^{\mathbf{h}}$ and $m_{\rm eff}^{\mathbf{A}}$ in Eqs. 10 and 11, are shown in Fig. 2. At the critical point $G(q) \propto q$, so that $\eta_{\mathbf{A}} = 1$. Note that, while $m_{\rm eff}^{\mathbf{A}}$ vanishes at $T = T_c$, $m_{\rm eff}^{\mathbf{h}}$ is finite but non-analytic. *As a result of the vortex loop blowout, the screening properties of the vortices are dramatically increased, and $m_{\rm eff}^{\mathbf{h}}$ increases sharply.*

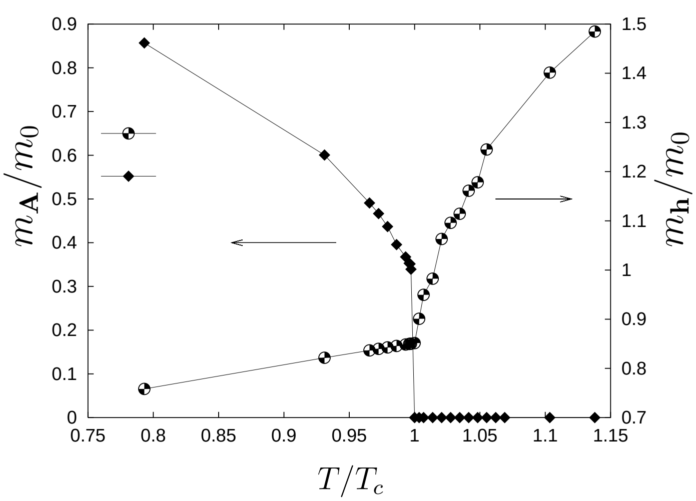

FIG. 2. $m_{\rm eff}^{\mathbf{A}}/m_0$ and $m_{\rm eff}^{\mathbf{h}}/m_0$ as functions of T.

To find $\eta_{\mathbf{h}}$ independently, we consider first the uncharged case $\lambda \to \infty$, $m_0 \to 0$. First, at an intermediate step in the transformation Eqs. 3 - 5, the action reads

$$S(\beta, e) = -\sum_{\mathbf{x}} \left\{ \frac{1}{2\beta}\mathbf{l}^2 + ei\mathbf{A}\cdot\mathbf{l} + \frac{1}{2}\left(\nabla\times\mathbf{A}\right)^2 \right\}. \quad (16)$$

Here, $\mathbf{l}$ is an integer field of closed current loops. Setting $e = 0$ in Eq. 5, the action of the dual Villain model is obtained,

$$\tilde{S}_V(\beta,\Gamma) = -\sum_{\mathbf{x}} \left\{ 2\pi i\mathbf{m}\cdot\mathbf{h} + \frac{1}{2\beta}\left(\Delta\times\mathbf{h}\right)^2 + \frac{\Gamma}{2}\mathbf{m}^2 \right\}. \quad (17)$$

Here, a term $\Gamma\mathbf{m}^2/2$ has been added, and $\tilde{S}_V(\beta,\Gamma)$ corresponds to the Villain-action in the limit $\Gamma \to 0$. However, it is physically reasonable to propose that the limit $\Gamma \to 0$ is non-singular, since the added term is short-ranged. It should therefore be an irrelevant perturbation, in renormalization group sense, to the long-ranged Biot-Savart interaction governing the fixed point, which is mediated by $\mathbf{h}$. Rescaling $\mathbf{h} \to \mathbf{h}e/2\pi$ in Eq. 17, we have[11] $Z(\beta, e) = \tilde{Z}_V\left(e^2/4\pi^2, 1/2\beta\right)$, leaving Eqs. 16 and 17 interchangeable; $\eta_{\mathbf{h}}$ from Eq. 17 should have the same value as $\eta_{\mathbf{A}}$ from Eq. 16. The above is demonstrated by our simulations based on Eqs. 6-9, *which are independent of the proposed form Eq. 17.*

To determine $\eta_{\mathbf{h}}$ we study the correlation function $\langle\mathbf{h_q h_{-q}}\rangle$ (Eq. 9) in the limit $m_0 \to 0$. At the uncharged fixed point of the original theory, which is the charged fixed point of the dual theory, we have $\lim_{q\to 0} 2\pi\beta^2 G(\mathbf{q}) = (1 - C_2(T))q^2 + ..., q^2 - C_3(T)q^{2+\eta_{\mathbf{h}}} + ...$, and $q^2 - C_4(T)q^4 + ...$, for $T < T_c, T = T_c$, and $T \geq T_c$, respectively. Here, $C_2(T)$ corresponds to the helicity modulus (superfluid density)[12], $C_3(T)$ is a critical amplitude, and $C_4(T)$ is the inverse of the mass of the dual gauge field for $T \geq T_c$. Correspondingly, we have $\lim_{q\to 0} < \mathbf{h_q h_{-q}} > = 2\beta C_2/q^2$, $2\beta C_3/q^{2-\eta_{\mathbf{h}}}$, and $2\beta C_4$, for $T < T_c, T = T_c$, and $T \geq T_c$, respectively. Note that $\mathbf{h}$ is massless for $T < T_c$, while it is massive for $T > T_c$, the dual system exhibits a "dual Meissner-effect" for $T \geq T_c$. At $T = T_c$, we have $q^2 \ \langle\mathbf{h_q h_{-q}}\rangle \simeq C_3(T)q^{\eta_{\mathbf{h}}}$. A plot of $q^2 \ \langle\mathbf{h_q h_{-q}}\rangle$ is shown in Fig. 3. A linear behaviour at $T = T_c$ is found, implying that $\eta_{\mathbf{h}} = 1$ when $e = 0$. Since $\eta_{\mathbf{h}} = 1$ in the uncharged case, this provides further support for the Hamiltonian Eq. 2.

We now set $e \neq 0$. The gauge field $\mathbf{h}$ becomes massive via the term $e^2\mathbf{h}^2/2$, which appears after integrating out the $\mathbf{A}$ field in Eq. 2. In this case, $\lim_{q\to 0}\langle\mathbf{h_q h_{-q}}\rangle = 2\beta/m_0^2$ from Eq. 9, and $\mathbf{h}(r)$ would naively have the trivial scaling dimension $(2-d)/2$. However, the mass term offers us a freedom in assigning dimensions to e and $\mathbf{h}$, by introducing renormalization Z-factors, here $e' = Z_{\mathbf{h}}^{1/2}e$ and $\mathbf{h}' = Z_{\mathbf{h}}^{-1/2}\mathbf{h}$.

Prior to integrating out $\mathbf{A}$ in Eq. 2, the mass appears in the term $ie\left(\nabla\times\mathbf{h}\right)\cdot\mathbf{A}$. Integration of the ϕ field, partial or complete, can only produce $(\nabla/i - e_d\mathbf{h})$-terms. In particular, this must hold during integration of fast Fourier-modes of the ϕ field. Thus, the term $i(\nabla\times\mathbf{h})\cdot\mathbf{A}$ *is renormalisation group invariant*, i.e. its prefactor must be dimensionless. In terms of scaled fields, at the charged fixed point of the original theory, we have $\mathbf{A}' = Z_{\mathbf{A}}^{-1/2}\mathbf{A}$, with $Z_{\mathbf{A}} \propto l^{\eta_{\mathbf{A}}}$, $\eta_{\mathbf{A}} = 1$[8]. For $\mathbf{h}$, we use $Z_{\mathbf{h}} \propto l^{\Delta}$, where Δ is *not* an anomalous scaling dimension ($\mathbf{h}$ is massive, cf. Fig. 2), but rather a contribution

to the engineering dimension of $\mathbf{h}$. Inserting this into the crossterm $ie\left(\nabla \times \mathbf{h}\right) \cdot \mathbf{A}$, we find the scaling dimension $\left(\eta_{\mathbf{A}} + \Delta\right)/2 - 1$, which must vanish. This gives the constraint $\Delta = 1$ to avoid conflicting results for $\eta_{\mathbf{A}}$.

Remarkably, therefore, the scaling dimension of $\mathbf{h}$ at $T = T_c$ is the same in both cases $m_0 = 0$ and $m_0 \neq 0$. The results for $\eta_{\mathbf{A}}$ and $\eta_{\mathbf{h}}$ in the previous paragraphs, are summed up in Table I.

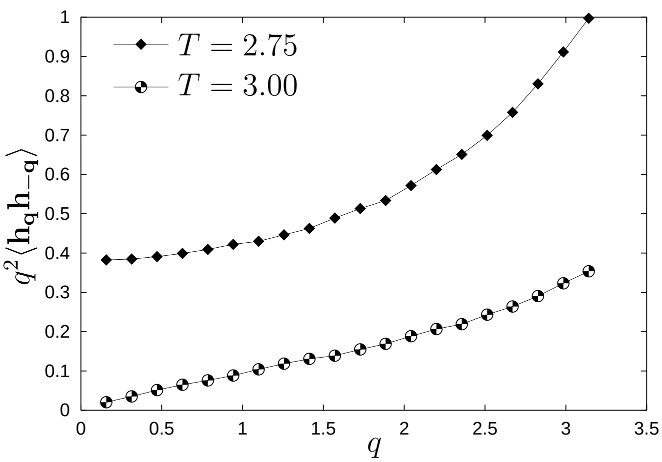

FIG. 3. $q^2\langle \mathbf{h_q h_{-q}} \rangle$ for two different T. For $\lambda = \infty$, $T_c = 3.00$.

We next consider the distribution of vortex loop sizes in the model Eq. 7, connecting the vortex loop distribution to the anomalous dimension of ϕ at T_c *both for the case* $e = 0$ *and* $e \neq 0$. During the simulations, we sample the distribution of loop-sizes $D(p)$, where p is the perimeter of a loop. This distribution function can be fitted to the form[5,4]

$$D(p) \propto p^{-\alpha} e^{-\beta p \varepsilon(T)}, \tag{18}$$

where $\varepsilon(T)$ is an effective line-tension for the loops. Figures showing the qualitative features of $D(p)$ can be found in Ref. 5. The critical point is characterised by a vanishing line-tension, and close to the critical point we find that $\varepsilon(T)$ vanishes as $\varepsilon(T) \propto |T - T_c|^{\gamma_\phi}$.

The vortex loops are the topological excitations of the GL and 3DXY models, at the same time they are the real-space representation of the Feynman diagrams of the dual field theory. By sampling $D(p)$, we obtain information about the dual field ϕ, particularly γ_ϕ can be identified as a *susceptibility* exponent for the ϕ field[5]. Using the scaling relation $\gamma_\phi = \nu_\phi\left(2 - \eta_\phi\right)$, and the important observation that even at the charged dual fixed point $\nu_\phi = \nu_{3DXY}$[5], this also gives us a value for the anomalous scaling dimension η_ϕ when we use the value $\nu_{3DXY} = 0.673$[13].

In Ref. 5 the vortex loops of the 3DXY model have been studied meticulously, yielding the value $\eta_\phi(0) = -0.18 \pm 0.07$. Since the dual of this model is isomorphic to a superconductor, $\eta_\phi(0)$ should be similar to $\eta_\psi(e)$ of the original GLT.

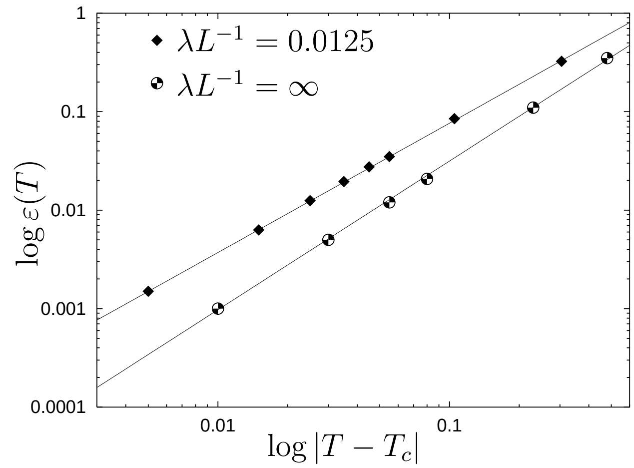

FIG. 4. $\ln \varepsilon(T)$ as a function of $\ln |T - T_c|$. The upper line shows the charged case with finite e, and the lower line shows the neutral case with $e = 0$. The slopes of the two straight lines are $\gamma_\phi = 1.315$ and $\gamma_\phi = 1.51$, corresponding to the anomalous dimensions $\eta_\phi = -0.24$ (neutral, i.e. dual charged) and $\eta_\phi = 0.04$ (charged, i.e. dual neutral), respectively.

We have studied the vortex loop distribution in both the neutral and the charged case. In the former case we find $\eta_\phi \simeq -0.24$, in good agreement with Ref. 5. In the latter case the dual theory has a $U(1)$ symmetry, and we would expect to find $\eta_\phi = \eta_{3DXY}$. The exponent η_{3DXY} has recently been determined with great accuracy to $\eta_{3DXY} = 0.038$[13], whereas we find $\eta_\phi \simeq 0.04$ which compares well with this value. Fig. 4 shows $\varepsilon(T)$ for both the charged and uncharged models. *It is evident that they belong to two different universality classes.*

In the case $e \neq 0$, *which corresponds to the dual neutral case*, the inverse ϕ-propagator is given by $G^{-1} = q^2 + \Sigma(q)$, where Σ is a self-energy, and $\Sigma(q) \sim q^{2-\eta}$ by definition. This gives a leading order behavior $G \sim 1/q^{2-\eta}$ provided $\eta > 0$, and we find $\eta = 0.04$ for this case. On the other hand, for the case $e = 0$, *which corresponds to the dual charged case*, dual gauge field fluctuations alter the physics, softening the long-wavelength ϕ field fluctuations. We obtain $G^{-1} = q^4 + \Sigma(q)$, again with $\Sigma(q) \sim q^{2-\eta}$, which now gives a leading order behavior $G \sim 1/q^{2-\eta}$, provided $\eta > -2$. Our result $\eta = -0.24$ for the case $e = 0$ (dual charged) is consistent with this, and also with the absolute bounds $\eta > 2 - D = -1$, in $D = 3$.

A consequence of the above is that in $D = 3$ dimensions, $\lambda \sim \xi^{(D-2)/(2-\eta_{\mathbf{A}})} = \xi$ at the charged critical point, in contrast to $\lambda \sim \sqrt{\xi}$ at the $3DXY$ neutral critical point. Since our results have been obtained directly by MC simulations, they are valid beyond all orders in perturbation theory.

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m_0	$\eta_{\mathbf{A}}$	FP, original theory	$\eta_{\mathbf{h}}$	FP, dual theory
0	0	Neutral 3DXY	1	Charged
Finite	1	Charged	0	Neutral 3DXY

TABLE I. Values of $\eta_{\mathbf{A}}$ and $\eta_{\mathbf{h}}$ at the stable neutral and charged critical points of the original and dual theories. FP is an abbreviation for fixed point.